Mandelbrot Set

Theory:

Start with a complex number (see interesting areas below) for k=0.

$C_k = u_k + iv_k$

Then iterate as follows:

$C_{k+1} = C_k^2 + C_0$

- if $C_\infty \to \infty$, then C_0 is outside the Mandelbrot set.
- if C_∞ = finite, then C_0 is inside the Mandelbrot set.

Practical Implementation:

- if $|C_k| \geq 2$, then C_∞ will become unbounded. Therefore, C_0 is outside the set, so stop iterating.
- if $|C_n| < 2$, then C_∞ will probably remain finite after infinite iteration. Therefore C_0 is inside the set, so stop iterating.

where, n is some limit on number of iterations (n is typically on the order of 1000-5000). The edge of the set becomes more sharply defined as n increases. n must be chosen (often by trial and error) depending upon the size of area in the complex being plotted.

$$
\begin{aligned}
C_{k+1} &= C_k^2 + C_0 \\
&= u_k^2 + 2iu_k v_k - v_k^2 + u_0 + iv_0 \\
&= (u_k^2 - v_k^2 + u_0) + i(2u_k v_k + v_0)
\end{aligned}
$$

So,

$u_{k+1} = u_k^2 - v_k^2 + u_0$ and

$v_{k+1} = 2u_k v_k + v_0$

- if $u_k^2 + v_k^2 \geq 4$, then C_∞ will become unbounded. Therefore, C_0 is outside the set, so stop iterating.
- if $u_{1000}^2 + v_{1000}^2 < 4$, then C_∞ will probably remain finite after infinite iteration. Therefore C_0 is inside the set, so stop iterating.

Colors are usually assigned to each point according to iteration results. Typically, if $|C_{1000}| < 2$ (i.e., likely included in set), the point is set to black. All other points are assigned colors depending on how many iterations it took to exceed a value of 2.

Entire Set:

Range of Real Component	Range of Imaginary Component
$-2 \le u_0 \le 0.5$	$-1.25 \le v_0 \le 1.25$

Interesting Areas:

Range of Real Component	Range of Imaginary Component
$0.26 \le u_0 \le 0.27$	$0 \le v_0 \le 0.01$
$-0.76 \le u_0 \le -0.74$	$0.01 \le v_0 \le 0.03$
$-1.26 \le u_0 \le -1.24$	$0.01 \le v_0 \le 0.03$
$-1 \le u_0 \le -0.5$	$-0.25 \le v_0 \le 0.25$
$-0.8 \le u_0 \le -0.7$	$0 \le v_0 \le 0.25$
$-0.75 \le u_0 \le -0.7$	$0.1 \le v_0 \le 0.25$
$-0.75 \le u_0 \le -0.7$	$0.2 \le v_0 \le 0.25$
$-0.74 \le u_0 \le -0.72$	$0.23 \le v_0 \le 0.25$
$-0.735 \le u_0 \le -0.73$	$0.237 \le v_0 \le 0.24$

Note: The visually pleasing effects of Mandelbrot plots are produced in the varying colors of points *surrounding* the set itself (assigned colors being based upon the number of iterations it took to exceed an absolute value of 2). Thus, areas of interest are always on the edges of the Mandelbrot set proper.